Solutions to Workbook-2 [Mathematics] | Permutation & Combination

JEE Archive	DAILY TUTORIAL SHEET 1

1.(B) The integer greater than 6000 may be of 4 digits or 5 digits. So, here two cases arise.
Case I When number is of 4 digits.
Four-digit number can start from 6, 7 or 8.

6, 7 *or 8*
↓
3	4	3	2

Thus, total number of 4-digit numbers, which are greater than
$6000 = 3 \times 4 \times 3 \times 2 = 72$.
Case II When number is of 5 digits.
Total number of five-digit numbers which are greater than 6000 = 5! = 120
$\therefore$ Total number of integers = 72 + 120 = 192

2.(C) $X — X — X — X — X$. The four digits 3, 3, 5, 5 can be arranged at (—) places in $\frac{4!}{2!\,2!} = 6$ ways.

The five digits 2, 2, 8, 8, 8 can be arranged at (X) places in $\frac{5!}{2!\,3!}$ ways = 10 ways.

Total number of arrangements = $6 \times 10 = 60$
(since events A and B are independent, therefore $A \cap B = A \times B$)

3.(B) Distinct n-digit numbers which can be formed using digits 2, 5 and 7 are 3^n. We have to find n, so that $3^n \geq 900 \Rightarrow 3^{n-2} \geq 100 \Rightarrow n - 2 \geq 5 \Rightarrow n \geq 7$, So, the least value of n is 7.

4.(C) Let n be the number of newspapers which are read by the students. Then, $60n = (300) \times 5 \Rightarrow n = 25$

5.(D) Since, the first 2 women select the chairs amongst 1 to 4 in ${}^4P_2$ ways. Now, from the remaining 6 chairs, three men could be arranged in ${}^6P_3$. $\therefore$ Total number of arrangements = ${}^4P_2 \times {}^6P_3$.

6.(A) Total number of five letter words formed from ten different letters = $10 \times 10 \times 10 \times 10 \times 10 = 10^5$
Number of five letters words having no repetition $= 10 \times 9 \times 8 \times 7 \times 6 = 30240$
$\therefore$ Number of words which have at least one letter repeated $= 10^5 - 30240 = 69760$

7.(B) Given, $T_n = {}^nC_3 \Rightarrow T_{n+1} = {}^{n+1}C_3$ $\quad\therefore\quad$ $T_{n+1} - T_n = {}^{n+1}C_3 - {}^nC_3 = 10$ (given)

$\Rightarrow {}^nC_2 + {}^nC_3 - {}^nC_3 = 10 \qquad \left[\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}\right]$

$\Rightarrow {}^nC_2 = 10 \quad\Rightarrow\quad n = 5$

8.(C) Here, ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3 = {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= \left({}^{47}C_4 + {}^{47}C_3\right) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$ [using ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$]

$= \left({}^{48}C_4 + {}^{48}C_3\right) + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3 = \left({}^{49}C_4 + {}^{49}C_3\right) + {}^{50}C_3 + {}^{51}C_3$

$= \left({}^{50}C_4 + {}^{50}C_3\right) + {}^{51}C_3 = {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$

9.(T) Let r consecutive integers be $x+1, x+2, \ldots\ldots, x+r$.

$\therefore \quad (x+1)(x+2)\ldots(x+r) = \frac{(x+r)(x+r-1)\ldots(x+1)x!}{x!} = \frac{(x+r)!}{(x)!} \cdot \frac{r!}{r!} = {}^{x+r}C_r \cdot (r)!$

Thus, $(x+1)(x+2)\ldots(x+r) = {}^{x+r}C_r \cdot (r)!$, which is clearly divisible by $(r)!$.

Hence, it is a true statement.

10.(C) Arrange the letters of the word COCHIN as in the order of dictionary CCHINO.
Consider the words starting from C.
There are 5! Such words. Number of words with the two C's occupying first and second place = 4!
Number of words starting with CH, CI, CN is 4! Each.
Similarly, number of words before the first word starting with CO = 4! + 4! + 4! + 4! = 96.
The word starting with CO found first in the dictionary is COCHIN. There are 96 words before COCHIN.

11.(A) Total number of arrangement of word BANANA = $\frac{6!}{3!\,2!} = 60$

The number of arrangements of words BANANA in which two N's appear adjacently = $\frac{5!}{3!} = 20$

Required number of arrangements = 60 – 20 = 40

12.(A) Since, $240 = 2^4 \cdot 3 \cdot 5$
We need even divisors of 240 that are not multiples of 4. Number of such divisors = The number of odd divisors because each divisor is of the form $2 \times$ odd divisor of 240. Number of odd divisors = 4.